Problems

Ted Eisenberg, Section Editor

****

This section of the Journal offers readers an opportunity to exchange interesting mathematical problems and solutions. Please send them to Ted Eisenberg, Department of Mathematics, Ben-Gurion University, Beer-Sheva, Israel or fax to: 972-86-477-648. Questions concerning proposals and/or solutions can be sent e-mail to <eisenbt@013.net>. Solutions to previously stated problems can be seen at <http://www.ssma.org/publications>.

Solutions to the problems stated in this issue should be posted before
Sept. 15, 2019

5547: *Proposed by Kenneth Korbin, New York, NY*

Given Heronian Triangle ABC with $\overline{AC} = 10201$ and $\overline{BC} = 10301$. Observe that the sum of the digits of $\overline{AC}$ is 4 and the sum of the digits of BC is 5. Find $\overline{AB}$ if the sum of its digits is 3.

(An Heronian Triangle is one whose side lengths and area are integers.)

5548: *Proposed by Michel Bataille, Reoun, France*

Given nonzero real numbers p and q, solve the system

$$\begin{cases} 2p^2x^3 - 2pqxy^2 - (2p-1)x = y \\ \\ 2q^2y^3 - 2pqx^2y + (2q+1)y = x \end{cases}$$

5549: *Proposed by Arkady Alt, San Jose, CA*

Let P be an arbitrary point in $\triangle\ ABC$ that has side lengths a, b, and c .

a) Find minimal value of

$$F(P) := \frac{a^2}{d_a(P)} + \frac{b^2}{d_b(P)} + \frac{c^2}{d_c(P)};$$

b) Prove the inequality $\dfrac{a^2}{d_a(P)} + \dfrac{b^2}{d_b(P)} + \dfrac{c^2}{d_c(P)} \geq 36r$, where r is the inradius.

5550: *Proposed by Ángel Plaza, University of the Las Palmas de Gran Canaria, Spain*

Prove that

$$\sum_{n=4}^{\infty}\sum_{k=2}^{n-2}\frac{1}{k\binom{n}{k}} = \frac{1}{2}.$$

5551: *Proposed by José Luis Díaz-Barrero, Barcelona Tech, Barcelona, Spain*